Narrow Annular Packed-Bed Radial Void Fraction Correlation

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The cylindrical packed bed—involving fluid flow, heat, and mass transfer—is presently the most preferred type of packed bed used in a variety of applied engineering and scientific fields. One particular configuration of the cylindrical packed bed is the annular packed bed (APB), made for compact catalytic reactors, column absorbers, and heat exchangers. Various studies involving a narrow annular packed bed (NAPB) $D_e/d \leq 4$ have been conducted. Wall heat-transfer coefficients and effective thermal conductivities for narrow annular beds have been investigated (Yagi and Kunii, 1960; Melanson and Dixon, 1985; Fedoseev and Shanin, 1986; Rimkevicius et al., 1991). Residence time distribution studies for flow in NAPBs have been reported (Kasthuri and Laddha, 1982), and hydraulic resistances and overall bed porosity have been determined for narrow annular packings (Rimkevicius et al., 1991; Kazazyan and Polyukhovich, 1997; Sodre and Parise, 1998). Also, non-Darcian effects on mixed convection in a vertical-packed sphere narrow annulus have been presented (Choi and Kulacki, 1993).

Hydrodynamic, heat and mass-transfer processes are strongly affected by the packed-bed structure in APBs. This is especially true for NAPBs, because the diameter aspect ratio is so small, $D_e/d \leq 4$. It is well understood that the wall in a cylindrical packed bed affects the void fraction distribution. Since NAPBs have two close walls that simultaneously affect the void fraction distribution, it is imperative to include this void fraction variation in transport models (Kazazyan and Polyukhovich, 1997; Sodre and Parise, 1998). To more accurately model the hydrodynamic, heat, and mass transfer in APBs, a usable correlation that closely predicts the void fraction distribution is essential. A radial void fraction correlation and profiles for APBs with $4 \leq D_e/d \leq 20$ have been reported (Mueller, 1999).

Thus, the purpose of this study is to calculate radial void fraction distributions of equal-sized spheres for narrow annular packed beds, $D_e/d \leq 4$, and to obtain a radial void fraction correlation from these distributions that can be used in NAPB transport models.

Void Fraction Development

No published data on radial void fraction distributions in NAPBs, $D_e/d \leq 4$ exist. The goal of void fraction development is to use analytical/numerical methods to acquire radial distributions. Mueller (1992) provides the detailed procedure and analytical equations used to determine the local radial void fraction for radial annular segments in cylindrical containers from which radial distributions can be obtained. The general concept in the analysis is to express the local radial void fraction in any radial annular segment in terms of the solid volume contributions acquired from spheres, with center coordinates at positions within a particle radius on either side of the radial annular segment. The packed bed of uniform spheres in a cylindrical container is divided into a large number of radial annular layers so that a detailed representation of the radial void fraction distribution can be produced. This procedure for determining the local void fraction that has been applied to APBs (Mueller, 1999) can also be used for NAPBs. In the case of NAPBs, only one type of intersecting geometry can occur for a sphere and the radial annular segment that intersects the sphere (Mueller, 1992). That geometry is for spheres with centers that are at radial locations greater than a particle radius from the center of the NABP. The other geometries described by Mueller (1992) do not apply in this investigation, because they are for spheres at locations less than a particle radius from the center of the container. This region does not exist in NAPBs.

The total volume of solids in a radial annular layer is the sum of the individual volume segments acquired from each sphere with a center coordinate that is at a location within a particle radius on either side of the layer. The void fraction for a radial annular layer is determined from the total volume of the layer and the total sum from the spheres in the layer. This process is performed for all the annular segments at a particular radial position to obtain the local radial void fraction. The result is an accurate void fraction distribution as a function of the radial position in the cylindrical packed bed (Mueller, 1992).

The procedure for calculating the radial void fraction distributions requires knowledge of the sphere center coordinates (Mueller, 1992). The center coordinates for this study of the equal-sized spheres in an NAPB are determined from an analytical/numerical method given by Mueller (1997). The method numerically constructs packed beds of identical spheres in right circular cylindrical containers by using a sequential packing model. The sequential packing model starts the packing from a base layer at the bottom of the cylindrical container. The base layer is determined from an algorithm (Mueller, 1997) that initially positions spheres next to each other, and next to the outer wall of the container. Spheres are added to the packing above the base layer using wall sphere positions or inner sphere positions (Mueller, 1997). Wall-sphere positions are locations where newly added spheres make contact with two other spheres and a wall. Inner sphere positions are locations where spheres make contact with three other spheres, but not with a wall. All sphere locations must be stable under gravity. The sequential model calculates the center coordinates of each newly added sphere. This method has been applied to APBs (Mueller, 1999), and, likewise, can be used for NAPBs. For this NAPB study, the Layer and Bennett models (Mueller, 1997) are used as the sequential packing procedures. Each procedure always selects the lowest vertical stable position to place a sphere. Spheres are placed on either two spheres and a wall (wall spheres) or on three spheres (inner spheres), as long as the position is a stable location (Mueller, 1997). The Layer model is applied to annular beds with diameter aspect ratios of $2 \leq D_e/d \leq 3.7$, and using only wall spheres for each layer. Layers above the base layer are alternated between the inner and outer walls of the annular packing. This is a similar packing that has been shown to occur for packed beds of low aspect ratios (Govindarao et al., 1992). The Bennett model is applied for diameter aspect ratios of $3.7 < D_e/d \leq 4$ where both wall and inner spheres are used. The center coordinates acquired from the sequential numerical procedures are used in the void fraction development to obtain the radial void fraction distributions.

Void Fraction Correlation

The radial void fraction distributions obtained from the void fraction development are used to formulate the radial void fraction correlation. The correlation is restricted to randomly packed beds in annular cylindrical containers of outside diameter D_o, inside diameter D_i, equivalent diameter D_e, and consisting of equal-sized spheres of diameter d, with diameter aspect ratios of $2 \leq D_e/d \leq 4$. The radial void fraction correlation is represented by the following principal equation

$$\epsilon_r = \epsilon_o + (1-\epsilon_o)\left\{J_o(\alpha r^*)e^{\beta(R^*-d^*)r^*} + J_o(\alpha[R^*-r^*])e^{\beta(R^*+d^*)(R^*-r^*)}\right\} \quad (1)$$

where

$$\alpha = 6.5 - 3.1e^{-2.5\ln^2 1.9(2-R^*)} \quad (2)$$

$$\beta = 8\cos(3\pi R^*)e^{-1.6R^*} - \frac{2}{R^*} \quad (3)$$

$$\epsilon_o = \frac{0.5-0.53R^*}{1-1.18R^*} \quad (4)$$

$$d^* = \frac{d}{D_o}, \quad 0 \leq d^* \leq \frac{1}{3} \quad (5)$$

$$r^* = \frac{r}{d}, \quad 0 \leq r^* \leq R^* \quad (6)$$

$$R^* = \frac{D_e}{2d}, \quad 1 \leq R^* \leq 2. \quad (7)$$

Results and Discussion

26 NAPB radial void fraction distributions with diameter aspect ratios ranging from $2 \leq D_e/d \leq 4$ were generated by the analytical/numerical methods reported by Mueller (1992, 1997). Some error is involved in applying these analytical/numerical equations resulting from the numerical integration technique needed to completely solve the equations. For this study, the Gauss-Kronrod method (Davis and Rabinowitz, 1984) is used for the numerical integration. Nevertheless, any high-order numerical integration algorithm will give results with errors that are so small as to be practically zero.

Mueller (1999) gave an empirical correlation that predicts the radial void fraction in annular packed beds for $4 \leq D_e/d \leq 20$. The general form of the principal equation given by the Mueller (1999) empirical correlation was initially used in the regression analysis to obtain the radial void fraction correlation given by Eq. 1. The principal equations used for annular packed beds with $4 \leq D_e/d \leq 20$ (Mueller, 1999) and $2 \leq D_e/d \leq 4$ (Eq. 1) involve a modified linear combination of damped Bessel functions formulated from an empirical correlation obtained by Mueller (1992).

In the evaluation process three criteria were adopted to assess the predictive usefulness of the final equation. First, in order to make the equation practical for transport modeling purposes, the equation had to include as many variables as feasible so that R^2 was as close as possible to the value of one. The R^2 coefficient for the correlation equation is a measure of the "goodness of fit" to that of the analytical/numerical distributions. The closer the value of the R^2 coefficient is to 1.0, the better is the statistical measure of the fit for the correlation. Second, to make the equation useful for predictive purposes, the number of variables was held to a minimum. Third, the one principal equation would have to closely predict the void fraction in the entire annular region between the two walls of the NAPB. All three criteria are satisfied by Eq. 1, and it is capable of modeling all 26 different radial void fraction distributions with R^2 values ranging between 90% to 99%.

In evaluating the coefficients, Eqs. 2–4, of the principal equation the above first two criteria were again used. In selecting the final equations for the coefficients, a compromise was used between the opposing requirements of the two criteria by selecting an equation for a variable that had a minimum number of terms, and still produced a reasonably high R^2 value. The first two coefficients, Eqs. 2 and 3, resolve the number and size of the peaks generated by the principal Eq. 1. The third coefficient, Eq. 4, determines the overall magnitude of the radial void fraction. Since the principal equation and the coefficients are empirically determined, it is inappropriate to associate any physical meaning to these coefficients.

Figure 1 shows three profiles of the 26 NAPB radial void fraction distributions generated from the regression analysis and the predicted curve from Eq. 1. The solid circular dots in

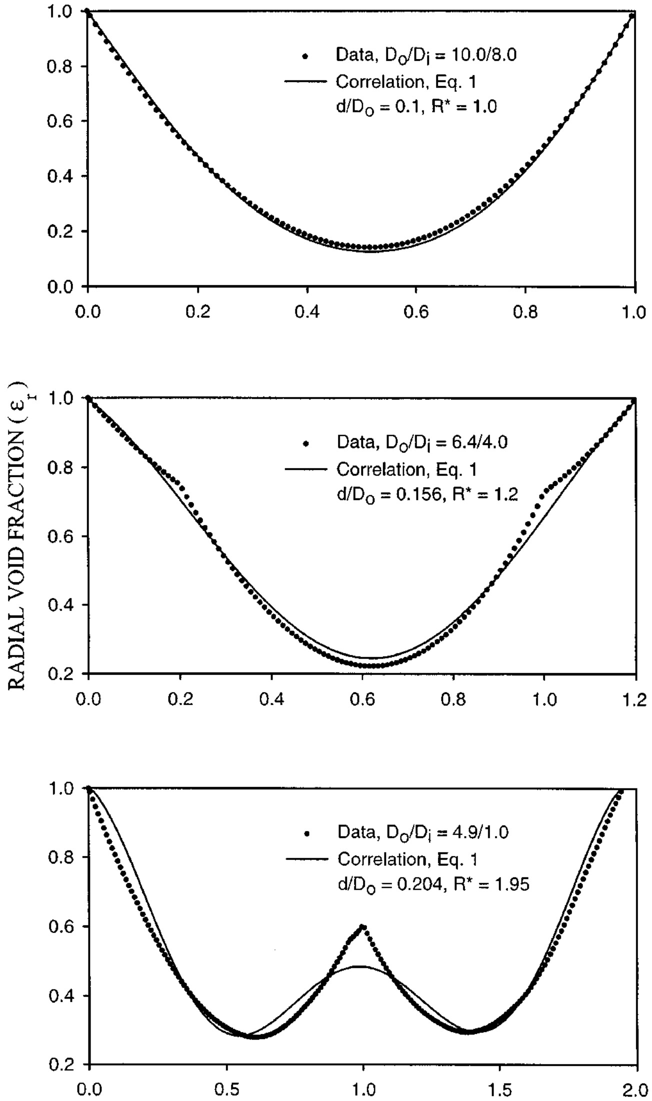

Figure 1. Narrow annular radial void fraction profiles.

Figure 1 represent the distributions obtained from the analytical/numerical method, (exact values can be obtained from the author) and the solid lines in the figure form the correlation given by Eq. 1. As shown, the two walls of the narrow annular bed simultaneously affect the radial void fraction distributions. The NAPB radial void fraction profiles are significantly different from those in high diameter aspect ratio annular beds (Mueller, 1999). While the latter profiles exhibit damped oscillations, the radial void fraction in the NAPB with $D_e/d = 2$ ($R^* = 1$) starts from unity at the outer wall, falls to a minimum about midpoint between the two walls, and then rises to reach unity at the inner wall. For $D_e/d = 4$ ($R^* = 2$) the profile begins at unity at the outer wall, falls to a minimum, then rises to a relative maximum about midpoint between the two walls, falls again to a minimum, and then rises to reach unity at the inner wall. The profiles with $2 < D_e/d < 4$ start from unity at each wall, fall with a kink in the curve to a minimum, and meet somewhere near the midpoint between the two walls. The positions of these kinks move from points near the walls to the center of the annulus as D_e/d increases from a value of two to four, with the kinks changing to relative maxima. Figure 1 shows one of these intermediate void fraction profiles and these kinks in the curve.

Because of the different curvatures of the inner and outer walls and the particular sphere packing that occurs in a NAPB, the radial void fraction distributions are not symmetrical between the two walls. This nonsymmetry becomes less pronounced for NAPBs with large bed diameters. However, the difference is relatively small and the correlation, in general, closely predicts the radial void fraction profiles for all cases.

Conclusion

In conclusion, this article presents several radial void fraction distributions for narrow annular packed beds, and a correlation equation that can closely predict the radial void fraction as a function of the radial position for diameter aspect ratios of $2 \leq D_e/d \leq 4$.

Notation

d = sphere diameter, m
d^* = dimensionless diameter, Eq. 5
D_e = equivalent diameter, $(D_o - D_i)$, m
D_i = annular bed inside diameter, m
D_o = annular bed outside diameter, m
J_o = Bessel function of the first kind of order zero
r = radial position from outside wall, m
r^* = dimensionless radial position from outside wall, Eq. 6
R^* = dimensionless radial thickness, Eq. 7

Greek Letters

α = coefficient, Eq. 2
β = coefficient, Eq. 3
ϵ_o = coefficient, Eq. 4
ϵ_r = radial void fraction, Eq. 1

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